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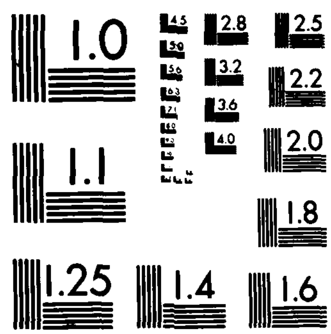

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Further Simplified Characterizations of Linear Complementarity Problems Solvable as Linear Program

by

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Abstract

Further Simplified Characterizations of Linear Complementarity Problems Solvable as Linear Programs

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Mangasarian developed necessary and sufficient conditions under which a linear complementarity problem can be solved as a linear program. These conditions include strict inequality systems which are difficult to apply. Using a bilinear programming approach and column scaling, we are able to derive further simplified conditions which replace the strict inequality systems with equations. Our simplified conditions yield new, checkable special cases of linear complementarity problems that are solvable as linear programs.

Key Words: Linear Complementarity Problem, Linear Programming, Bilinear Programming

Abbreviated Title: Conditions for Solving LCP as LP

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Further Simplified Characterizations of Linear Complementarity Problems Solvable as Linear Programs

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1. Introduction

In a series of papers [3,4,5,6], Mangasarian investigated necessary and sufficient conditions under which the linear complementarity problem (LCP) of finding a real n-vector x such that

$$Mx + q \geq 0,\ x \geq 0,\ x^T(Mx + q) = 0 \qquad (1)$$

is solvable (has a nonempty solution set), where M and q are given real n×n and n×1 matrices, respectively. For specified nonnegative vectors r and s that appear in these conditions, the linear program (LP)

$$\underset{x}{\text{minimize}}\ (r^T+s^TM)\,x$$

$$\text{subject to } Mx + q \geq 0,\ x \geq 0 \qquad (2)$$

has a nonempty optimal solution set which is a subset of the LCP's solution set (see [5] for details). Thus, such conditions characterize LCP's that can be solved as LP's. In a follow-up paper [6], simpler characterizations were derived.

Aside from the obvious value of being able to solve some LCP's (which are NP-complete) as LP's (which are polynomially solved), such conditions may be useful in developing constructive characterizations of matrix classes that possess complementary solutions (see, e.g., [1]),

especially subclasses that are not independent of the vector q. In this note, we use a bilinear programming approach to give a new, more direct derivation of conditions that further simplify the characterizations in [6]. We use our conditions to derive new special cases under which an LCP is solvable as an LP. In particular, unlike most of the cases in [5], our simplified characterizations allow for new cases that incorporate the vector q.

The following notation is used in this paper. The space of $m \times n$ real matrices is denoted by $R^{m \times n}$. We write $R^{m \times 1} = R^m$ and use a superscript T to transpose all matrices. The vector in R^n having ones for all of its elements is denoted by e and $I \in R^{n \times n}$ is the usual identity matrix. A Z-matrix is a square matrix with nonpositive off-diagonal elements and Z is used to denote the class of such matrices. The class $K \subset Z$ consists of nonsingular Z-matrices whose inverses are nonnegative.

2. Previous Work

The results of Mangasarian [6] are summarized in this section. Interested readers should consult the source for full proofs.

The dual program (DP) of the linear program (2) is defined as

$$\begin{aligned} &\underset{y}{\text{maximize}} \quad -q^T y \\ &\text{subject to } -M^T y + r + M^T s \geq 0, \quad y \geq 0. \end{aligned} \tag{3}$$

This problem plays an important role in proving the next three theorems which characterize the conditions under which an LCP can be solved as an LP.

<u>Theorem 2.1</u> The LCP has a solution if and only if the LP is solvable for some $r,s \in R^n$, $r,s \geq 0$, such that an associated dual optimal variable y satisfies

$$(I-M)^T y + r + M^T s > 0.$$

Furthermore, each solution of the LP solves the LCP.

<u>Theorem 2.2</u> If there exist $c,r,s \in R^n$ and $Z_1,Z_2 \in R^{n\times n}$ such that

$$MZ_1 = Z_2 + qc^T,\ r^T Z_1 + s^T Z_2 > 0,\ c,r,s \geq 0,\ Z_1,Z_2 \in Z, \tag{4}$$

then the feasible LCP has a solution which can be obtained by solving the LP. Conversely, if the LCP has at least one solution x which is a vertex of $\{x: Mx + q \geq 0,\ x \geq 0\}$ which is also nondegenerate, that is, $x + Mx + q > 0$, then conditions (4) are satisfied and each solution of the LP solves the LCP.

<u>Theorem 2.3</u> The LCP has a solution if and only if the LP is solvable for some $r,s \in R^n$ which must satisfy the following conditions:

$$\begin{aligned} &(a)\quad MZ_1 = Z_2 + qc^t \\ &(b)\quad r^T Z_1 + s^T Z_2 \geq 0 \\ &(c)\quad r^T Z_1 + s^T Z_2 + c^T > 0 \end{aligned} \tag{5}$$

$$\text{(d)} \quad r + s \qquad > 0$$

$$c, r, s \geq 0, \quad Z_1, Z_2 \in Z,$$

for some vector $c \in R^n$ and some matrices $Z_1, Z_2 \in R^{n\times n}$. Furthermore, each solution of the LP solves the LCP.

By examples Mangarasian [6] shows that the nondegenerate vertex assumption in Theorem 2 cannot be relaxed. Theorem 1 strengthens the sufficient conditions of [3, Lemma 1] to be necessary. Theorem 2 simplifies the sufficient conditions of [4, Theorem 1] and strengthens them to be also necessary under the nondegeneracy assumption. Theorem 3 simplifies the sufficient conditions of [5, Theorem 2] which were shown to be also necessary by Pang [7].

3. Some Simplified Characterizations

The theorems in the preceding section give conditions under which appropriate nonnegative vectors r and s exist such that the solution of the linear program (2) will also solve the LCP. If r and s are allowed to vary in (2), the natural question to ask is under what conditions will minimizing over these variables along with x produce a solution to the LCP. Of course, in this situation the objective function of (2) is bilinear. The answer to the question turns out to be surprisingly simple.

Consider the following bilinear program (BLP)

$$\underset{(x,r,s)}{\text{minimize}} \; (r^T + s^T M)x + q^T s$$

$$\text{subject to} \quad Mx + q \geq 0,\ x \geq 0 \qquad (6)$$
$$r + s = e,\ r \geq 0,\ s \geq 0.$$

Notice that for fixed (r,s) the bilinear program (6) reduces to the linear program (2) (ignoring the feasibility conditions on r and s above.) In addition, we now also have a linear program in (r,s) for fixed x.

<u>Lemma 3.1</u> The LCP has a solution if and only if the BLP has a vanishing optimal objective value.

<u>Proof</u>: Rewrite (6) as the bilinear program

$$\underset{(x,s)}{\text{minimize}}\ (e-s)^T x + s^T(Mx + q)$$

$$\text{subject to } Mx + q \geq 0,\ x \geq 0 \qquad (7)$$
$$0 \leq s \leq e.$$

The proof follows from the observation that the objective function is nonnegative over the feasible region. (See Appendix for details.)

For fixed s, the bilinear program (7) becomes the linear program

$$\underset{x}{\text{minimize}}\ (e-s + M^Ts)^T x + q^T s \qquad (8)$$

$$\text{subject to } Mx + q \geq 0,\ x \geq 0.$$

The dual of (8) is the linear program

$$\underset{y}{\text{maximize}} \quad -q^T(y-s)$$

$$\text{subject to} \quad M^T(y-s) < e-s, \; y > 0.$$

Under the transformation $y = u + s$, this is equivalent to the problem

$$\underset{u}{\text{maximize}} \quad -q^T u \qquad (9)$$

$$\text{subject to } M^T u < e-s, \; u > -s.$$

When the roles of (2) and (3) in the results of Mangasarian [6] are assumed by (8) and (9) above, the three theorems of the preceding section can be further simplified. By column scaling we are also able to replace the strict inequalities in (4) and (5c) by equations. Proofs for the next three theorems are provided in the Appendix.

<u>Theorem 3.2</u> The LCP has a solution if and only if the linear program (8) is solvable for some $s \in R^n$, $0 < s < e$, such that an associated dual optimal variable u satisfies

$$(M-I)^T u < e. \qquad (10)$$

Furthermore, each solution of the linear program (8) solves the LCP.

<u>Theorem 3.3</u> If there exist $c, s \in R^n$ and $Z_1, Z_2 \in R^{n\times n}$ such that

$$\begin{aligned}
&\text{(a)} \quad MZ_1 = Z_2 + qc^T \\
&\text{(b)} \quad (e-s)^T Z_1 + s^T Z_2 = e^T \qquad\qquad (11) \\
&\qquad c \geq 0,\ 0 \leq s \leq e,\ Z_1, Z_2 \in Z,
\end{aligned}$$

then the feasible LCP has a solution which can be obtained by solving the linear program (8). Conversely, if the LCP has at least one solution x which is a vertex of $\{x\colon Mx + q \geq 0,\ x \geq 0\}$ which is also nondegenerate, that is, $x + Mx + q > 0$, then the conditions (11) are satisfied and each solution of the linear program (8) solves the LCP.

<u>Theorem 3.4</u> The LCP has a solution if and only if the linear program (8) is solvable for some $s \in R^n$ which must satisfy the following conditions:

$$\begin{aligned}
&\text{(a)} \quad MZ_1 = Z_2 + qc^t \\
&\text{(b)} \quad (e-s)^T Z_1 + s^T Z_2 + c^T = e^T \qquad\qquad (12) \\
&\qquad 0 \leq c \leq e,\ 0 \leq s \leq e,\ Z_1, Z_2 \in Z,
\end{aligned}$$

for some vector $c \in R^n$ and some matrices $Z_1, Z_2 \in R^{n\times n}$. Furthermore, each solution of the linear program (8) solves the LCP.

In addition to the cases in [2] and [5], new special cases can be derived from the conditions (11) and (12). Let $p = e+(M-I)^T s$ denote the objective cost vector and let $S = \{x: Mx+q \geq 0, x \geq 0\}$ denote the feasible set of the linear program (8). The following theorem provides new cases such that the LCP is solvable as a linear program. In particular, the simplified conditions (11) and (12) allow for the easy incorporation of the vector q into our cases.

<u>Theorem 3.5</u> Let $S \neq \emptyset$, and let (M,q) satisfy any of the conditions below. Then the linear complementarity problem (1) has a nonempty solution set, at least one element of which can be obtained by solving the linear program (8) with the p indicated.

(a) $M-qc^t \varepsilon Z,\ 0 \leq c \leq e,\ (I-M+qc^T)^T s = c,\ 0 \leq s \leq e,\ p = e+(M-I)^T s.$

(b) $M-qc^T \varepsilon Z,\ c \geq 0,\ (I-M+qc^T)^T s = 0,\ 0 \leq s \leq e,\ p = e+(M-I)^T s.$

(c) $-M-qc^T \varepsilon Z,\ 0 \leq c \leq e,\ (I-M-qc^T)^T s = 2e-c,\ 0 \leq s \leq e,$
$p = e+(M-I)^T s.$

(d) $-M-qc^T \varepsilon Z,\ c \geq 0,\ (I-M-qc^T)^T s = 2e,\ 0 \leq s \leq e,\ p = e+(M-I)^T s.$

(e) $MZ_1 = Z_2+qc^T,\ Z_1 \varepsilon Z,\ Z_2 \varepsilon Z,\ Z_1^T e = e,\ c \geq 0,\ p = e.$

(f) $MZ_1 = Z_2+qc^T,\ Z_1 \varepsilon Z,\ Z_2 \varepsilon Z,\ Z_1^T e = e-c,\ 0 \leq c \leq e,\ p = e.$

(g) $MZ_1 = Z_2+qc^T,\ Z_1 \varepsilon Z,\ Z_2 \varepsilon Z,\ Z_2^T e = e,\ c \geq 0,\ p = M^T e.$

(h) $MZ_1 = Z_2+qc^T,\ Z_1 \varepsilon Z,\ Z_2 \varepsilon Z,\ Z_2^T e = e-c,\ 0 \leq c \leq e,\ p = M^T e.$

(i) $(M-I)^{-1} \varepsilon Z,\ 0 \leq e^T (M-I)^{-1} \leq e^T,\ p = (M-I)^T e.$

(j) $(I-M)^{-1} \varepsilon Z,\ e^T \leq e^T (I-M)^{-1} \leq 2e^T,\ p = (I-M)^T e.$

<u>Proof</u>: (a) In Theorem 3.4, set $Z_1 = I$ and $Z_2 = M-qc^T$.

(b) In Theorem 3.3, set $Z_1 = I$ and $Z_2 = M-qc^T$.

(c) In Theorem 3.4, set $Z_1 = -I$ and $Z_2 = -M-qc^T$.

(d) In Theorem 3.3, set $Z_1 = -I$ and $Z_2 = -M-qc^T$.

(e) In Theorem 3.3, set $s = 0$.

(f) In Theorem 3.4, set $s = 0$.

(g) In Theorem 3.3, set $s = e$.

(h) In Theorem 3.4, set $s = e$.

(i) In Theorem 3.3, set $Z_1 = (M-I)^{-1}$, $Z_2 = I+(M-I)^{-1}$, $c = 0$, and $s^T = e^T[I-(M-I)^{-1}]$.

(j) In Theorem 3.3, set $Z_1 = (I-M)^{-1}$, $Z_2 = (I-M)^{-1}-I$, $c = 0$, and $s^T = e^T[(I-M)^{-1}-I]$.

Remark. A given LCP may fall in more than one case above, each of which could lead to a different complementary solution. Cases (a) and (b) are not subsets of each other since we can construct problems that fall in either and in both. The same holds for cases (c) and (d), (e) and (f), and (g) and (h). In part (b), we can always take $s=0$, but other values may work in some instances so alternative solutions could be found. Excluding (e) through (h), the remaining cases are easily checkable. Other simplified conditions can be derived from the above theorem as illustrated by the following corollaries.

Corollary 1 Let $S \neq \emptyset$, and let M satisfy the conditions below. Then the LCP (1) has a solution that can be obtained by solving the LP (8) with the p indicated.

(a) $M \epsilon Z$, $p = e$.

(b) $M^{-1} \varepsilon Z$, $p = M^T e$.

(c) $I - M \varepsilon K$, $0 < e^T (M-I)^{-1} < e^T$, $p = (M-I)^T e$

(d) $M - I \varepsilon K$, $e^T < e^T (I-M)^{-1} < 2e^T$, $p = (I-M)^T e$

(e) $-M \varepsilon Z$, $(I-M)^T s = 2e$, $0 < s < e$, $p = e + (M-I)^T s$.

(f) $-M \varepsilon Z$, $-M^T e = e$, $p = M^T e$

<u>Proof</u>:

(a) In parts (a) and (b) of Theorem 3.5, set $s = c = 0$. Also can be obtained from parts (e) and (f) by setting $c = 0$ and $Z_1 = I$.

(b) Assume M is nonsingular and set $Z_2 = I$ and $c = 0$ in either parts (g) or (h) of Theorem 3.5.

(c) Subsumed by part (i) of Theorem 3.5.

(d) Subsumed by part (j) of Theorem 3.5.

(e) In parts (c) or (d) of Theorem 3.5, set $c = 0$.

(f) In parts (g) or (h) of Theorem 3.5, set $Z_1 = -I$ and $c = 0$.

<u>Corollary 2</u> Let $S \neq \emptyset$, and let (M,q) satisfy the conditions below. Then the LCP (1) has a solution that can be obtained by solving the LP (8) with the p indicated.

(a) $M = -qe^T$, $p = M^T e$.

(b) $M = qs^T$, $0 < s < e$, $p = e + (M-I)^T s$.

(c) $M = qc^T$, $c > 0$, $p = e$.

(d) $M = -I - qc^T$, $c > 0$, $p = M^T e$.

(e) $M = -I + qc^T$, $c > 0$, $p = e$.

(f) $M = I + qc^T$, $c > 0$, $p = e + (M-I)^T s$, where $0 < s < e$.

(g) $M - qc^T \varepsilon Z$, $c > 0$, $p = e$.

(h) $M^{-1}(I + qc^T) \varepsilon Z$, $c > 0$, $p = M^T e$.

<u>Proof</u>: In Theorem 3.5, set:

(a) $Z_1 = -I$, $Z_2 = 0$, and $c=e$ in part (h).

(b) $c = s$ in part (a).

(c) $Z_1 = I$ and $Z_2 = 0$ in part (e).

(d) $Z_1 = -I$ and $Z_2 = I$ in part (g).

(e) $Z_1 = I$ and $Z_2 = -I$ in part (e).

(f) $I-M+qc^T = 0$ in part (b).

(g) $s = 0$ in part (b).

(h) $Z_1 = M^{-1}(I+qc^T)$ and $Z_2 = I$ in part (g).

<u>Remark</u>. Cases (a) and (b) of Corollary 1 are well-known. They are also special cases of (g) and (h), respectively, in Corollary 2. Notice that $M-qc^T \epsilon Z$ for some $c > 0$ if and only if for each $j = 1,\ldots,n$, either $M_{ij} < 0$ for all $i \neq j$ or $M_{ij} < c_j q$ for all $i \neq j$ and some $c_j > 0$, where M_{ij} is the (i,j)-element of M and c_j is the <u>jth</u> component of c. In other words, excluding the diagonal elements, if every column of M is either nonpositive or does not exceed a positive multiple of q, then a solution of the associated LCP can be obtained by solving a linear program with objective cost vector e. In particular, a problem does <u>not</u> satisfy this condition if an off-diagonal element of M is positive and the element of q in the same row is nonpositive. On the other hand, we can easily characterize all vectors q that do satisfy this condition.

Appendix

Proof of Lemma 3.1

Suppose $\bar{x}$ solves the LCP and let $\bar{s}$ denote the n-vector whose ith component is defined as

$$\bar{s}_i = \begin{cases} 1 & \text{if } \bar{x}_i > 0 \\ 0 & \text{if } \bar{x}_i = 0 \end{cases} \tag{13}$$

Clearly, $(\bar{x},\bar{s})$ solves (7) with a vanishing optimal objective value.

Now suppose that $(\bar{x},\bar{s})$ solves (7) and the objective function vanishes at this point. Then

$$(1-\bar{s}_i)\bar{x}_i = 0 \quad \text{and} \quad \bar{s}_i(M\bar{x} + q)_i = 0$$

for all $i = 1,2,\ldots,n$, which implies that $\bar{x}^T(M\bar{x} + q) = 0$.

Proof of Theorem 3.2

Suppose $\bar{x}$ solves the LCP and let $\bar{s}$ be specified by (13). By Lemma 3.1, $\bar{x}$ solves (8) for $s = \bar{s}$ and $q^T\bar{u} = 0$ for all optimal solutions $\bar{u}$ of (9). In particular, $\bar{u} = 0$ solves (9) and satisfies (10).

For some $0 < \bar{s} < e$, suppose that $\bar{x}$ solves (8) and its complementary dual solution $\bar{u}$, which solves (9), satisfies (10). By complementary slackness, we have

$$\bar{x}^T(e-\bar{s} - M^T\bar{u}) = 0 \quad \text{and} \quad (\bar{u}+\bar{s})^T(M\bar{x} + q) = 0.$$

If $\bar{x}$ does not solve the LCP, then there is an i such that $\bar{x}_i(M\bar{x} + q)_i > 0$ so that

$$(e-\bar{s} + M^T\bar{u})_i = 0 \text{ and } (\bar{u} + \bar{s})_i = 0.$$

Adding gives $[e-(I-M)^T\bar{u}]_i = 0$ which contradicts the assumption that $\bar{u}$ satisfies (10). Hence, $\bar{x}$ solves the LCP. To show the second part of the theorem, let $(\bar{x}, \bar{u}, \bar{s})$ be as above and let $\hat{x} \neq \bar{x}$ also solve (8) for $s = \bar{s}$. By the strong duality theorem we have

$$\begin{aligned} 0 &= (e-\bar{s}+M^T\bar{s})^T\hat{x} + q^T\bar{s} + q^T\bar{u} + (M^T\bar{u})^T\hat{x} - (M^T\bar{u})^T\hat{x} \\ &= (e-\bar{s} - M^T\bar{u})^T\hat{x} + (\bar{u}+\bar{s})^T(M\hat{x} + q). \end{aligned}$$

Since each term is nonnegative, we must have

$$\hat{x}^T(e - \bar{s} - M^T\bar{u}) = 0 \quad \text{and} \quad (\bar{u}+\bar{s})^T(M\hat{x} + q) = 0.$$

Because $\bar{u}$ satisfies (10), we conclude by our argument above that $\hat{x}$ solves the LCP. Notice that the complementary dual solution of $\hat{x}$ need not satisfy (10).

Proof of Theorem 3.3

Assume the LCP is feasible, let $(\tilde{s}, \tilde{c}, \tilde{Z}_1, \tilde{Z}_2)$ solve (11) and let $\tilde{x}$ solve (8) for $s = \tilde{s}$. Then $\tilde{x}$ solves (2) for $(\tilde{r}, \tilde{s})$, where $\tilde{r} = e - \tilde{s}$, and $(\tilde{r}, \tilde{s}, \tilde{c}, \tilde{Z}_1, \tilde{Z}_2)$ solves (4). Hence, by Theorem 2.2, the LCP has a

solution that can be obtained by solving (2), which is equivalent to (9) when $r = e - s$.

Now suppose $\bar{x}$ is a nondegenerate extreme point solution of the LCP and let $\bar{s}$ be specified by (13). By Lemma 3.1, $\bar{x}$ solves (8) for $s = \bar{s}$ with vanishing optimal objective value. Hence, $\bar{u} = 0$ is the unique solution of (9). Since $\bar{u}$ satisfies (10), by Theorem 3.2, each solution of (8) solves the LCP. Moreover, conditions (11) are satisfied since, following the proof of Theorem 2.2 in [6], they are equivalent to

$$\left.\begin{aligned} M^T u &\leq e - s \\ u &\geq -s \\ q^T u &\leq 0 \end{aligned}\right\} \Rightarrow (M - I)^T u < e$$

for some $o \leq s \leq e$, which is clearly satisfied by $(\bar{s}, \bar{u})$ above.

<u>Proof of Theorem 3.4</u>

Suppose $\bar{x}$ solves the LCP and let $\bar{s}$ be specified by (13). By Lemma 3.1, $\bar{x}$ solves (8) for $s = \bar{s}$. Let

$$\bar{c} = e, \ \bar{Z}_1 = -\bar{x}e^T \text{ and } \bar{Z}_2 = -(M\bar{x} + q)e^T.$$

Then $(\bar{s}, \bar{c}, \bar{Z}_1, \bar{Z}_2)$ satisfies (12).

Now let $(\tilde{s}, \tilde{c}, \tilde{Z}_1, \tilde{Z}_2)$ satisfy (12) and let $\tilde{x}$ solve (8) for $s = \tilde{s}$. With $\tilde{r} = e - \tilde{s}$, it is easy to verify that $(\tilde{r}, \tilde{s}, \tilde{c}, \tilde{Z}_1, \tilde{Z}_2)$ satisfies (5) and $\tilde{x}$ solves (2) for $(\tilde{r}, \tilde{s})$. Hence, by Theorem 2.3, the LCP has a solution and each solution of (2), which is equivalent to (8) when $r = e - s$, solves the LCP.

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